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9231/22

October/November 2022

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 (a)** Find the set of values of k for which the system of equations

$$x + 2y + 3z = 1,$$

$$kx + 4y + 6z = 0,$$

$$7x + 8y + 9z = 3,$$

has a unique solution.

[3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Interpret the situation geometrically in the case where the system of equations does not have a unique solution. [2]

[illegible]

2 A curve has equation

$$(x+1)y+y^2=2.$$

- (a) Show that $\frac{dy}{dx} = -\frac{2}{3}$ at the point $(0, -2)$. [3]

[illegible]

- (b)** Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, -2)$. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 3 (a)** A curve has equation $y = e^x + \frac{1}{4}e^{-x}$, for $0 \leq x \leq 1$. Find, in terms of π and e , the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [6]

[illegible]

- (b) Using standard results from the list of formulae (MF19), or otherwise, find the Maclaurin's series for $e^x + \frac{1}{4}e^{-x}$ up to and including the term in x^2 . [2]

4 Find the solution of the differential equation

$$(4t^2 - 1) \frac{dx}{dt} + 4x = 4t^2 - 1$$

for which $x = 3$ when $t = 1$. Give your answer in the form $x = f(t)$.

[9]

[illegible]

5 (a) Write down the fourth roots of unity.

[1]

(b) Use de Moivre's theorem to show that

$$\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1.$$

[4]

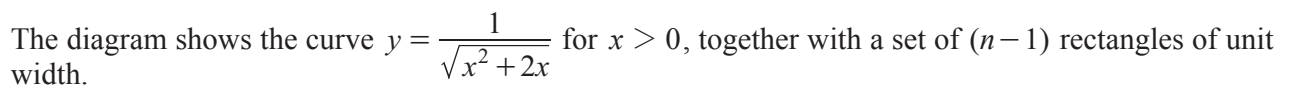
(c) Hence obtain the real roots of the equation

$$16(8x^4 - 8x^2 + 1)^4 - 9 = 0$$

in the form $\cos(q\pi)$, where q is a rational number.

[5]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.


$$\sum_{r=1}^n \frac{1}{\sqrt{r^2 + 2r}} < \ln(n+1 + \sqrt{n^2 + 2n}) + \frac{1}{3}\sqrt{3} - \ln(2 + \sqrt{3}). \quad [10]$$
This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$ for which $\mathbf{e}$ is a corresponding eigenvector. [2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 15 & -4 & 3 \\ 3 & 0 & 2 \end{pmatrix}.$$

- (b) Given that -1 is an eigenvalue of $\mathbf{A}$, find a corresponding eigenvector. [2]

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- (c) It is also given that $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ are eigenvectors of $\mathbf{A}$. Find the corresponding eigenvalues. [2]

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(e) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are rational numbers to be determined. [4]

[Turn over

- 8 It is given that $y = \cosh u$, where $u > 0$, and

$$\sqrt{\cosh^2 u - 1} \left(\frac{d^2 u}{dx^2} + \frac{du}{dx} \right) + \cosh u \left(\frac{du}{dx} \right)^2 - 2 \cosh u = 4e^{-x}.$$

- (a) Show that

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 4e^{-x}. \quad [4]$$

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- (b) Find u in terms of x , given that, when $x = 0$, $u = \ln 3$ and $\frac{du}{dx} = 3$. [10]

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